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ON A CONVERGING SPHERICAL FLAME FRONT

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Abstract—The paper deals with the frontal propagation velocity of a spherically symmetric flame converging to a focussing center. Such a problem can describe burning in a disunited zone in the laminar theory of a turbulent flame. It is shown that at Lewis number exceeding unity, the temperature in the reaction zone increases together with propagation velocity as the front approaches the center. By contrast, at Lewis numbers below unity, propagation is slowed down and the temperature in the reaction zone increases to extinction.

The problem is solved under the assumption of strong temperature-dependence for the reaction rate.

The asymptotic approach developed in solving this problem can then be applied to investigate the normal flame propagation stability. The article intends to show that at Lewis number exceeding unity propagation is unstable and a self-oscillation regime sets in.

NOMENCLATURE

A, see equation (1.5);
C, dimensionless concentration, referred to C_0;
C_0, initial concentration of unburned gas;
c_p, specific heat;
E, activation energy;
F, TW;
G, see equation (2.21);
L, Lewis number;
l_T, thermal thickness of flame ($= \kappa_b/U_b\rho_b c_p$);
N, dimensionless activation energy ($= E/R^0T_b$);
n, order of reaction;
Q, strength of surface source;
R^0, universal gas constant;
r, dimensional radial coordinate, referred to l_T;
r_f, coordinate of flame front;
S, see equation (4.5);
T, dimensionless temperature, referred to T_b;
T_0, temperature of unburned gas;
T_b, adiabatic temperature of burned gas;
t, dimensionless time, referred to l_T/U_b;
U, dimensionless radial component of gas velocity, referred to U_b;
U_b, normal velocity of flame;
W, see equation (1.4);
x, see equation (2.3);
X, see equation (4.4);
z, dimensionless frequency factor, referred to z_b;
z_b, frequency factor at T_b.

Greek symbols

δ, Dirac function;
ε, thermal expansion coefficient of gas ($= T_0/T_b$);
θ, see equation (4.5);
ξ, see equation (2.21);
ρ, dimensionless density, referred to ρ_b;
ρ_b, density of combustion products;
τ, Φ, see equation (2.4);
κ, dimensionless thermal conductivity, referred to κ_b;
κ_b, thermal conductivity of combustion products;
λ, κ/T;
ζ, see equation (5.8).

1. FORMULATION OF PROBLEM AND FUNDAMENTAL EQUATION

CONSIDER spherically-symmetric motion, whereby a combustion wave in a combustible gas mixture at constant initial temperature T_0 and concentration of the combustible component C_0 converges to the center of symmetry (Fig. 1). Such a wave is generated, for example, by a spherical source imparting to the mixture

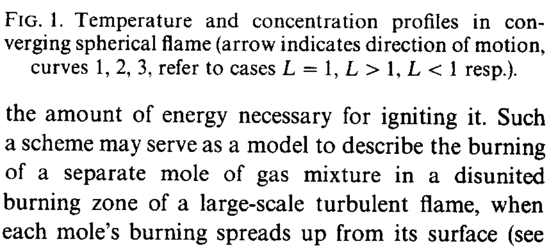

FIG. 1. Temperature and concentration profiles in converging spherical flame (arrow indicates direction of motion, curves 1, 2, 3, refer to cases $L = 1$, $L > 1$, $L < 1$ resp.).

the amount of energy necessary for igniting it. Such a scheme may serve as a model to describe the burning of a separate mole of gas mixture in a disunited burning zone of a large-scale turbulent flame, when each mole's burning spreads up from its surface (see the laminar theory of a turbulent flame [1]).

As the front converges, its curvature increases continuously, with the attendant changes in the process taking place in the reaction zone. The latter, in turn, governs the variation of the propagation velocity of the flame.

We are concerned with the motion of the front at distance sufficiently remote from the heat source, where the initial conditions are substantially "forgotten" and the front has settled down to a certain limiting regime, which we wish to determine. As is known, the flame represents a wave involving diffusion and conduction, accompanied by release of chemical energy in the form of heat and thermal expansion of the gas. In the case of *similarity* of the temperature and concentration fields ($L = 1$), combustion proceeds with a constant temperature at the front [2]. Here, the only characteristic parameter outside the reaction zone is the thermal thickness of the flame l_T, which also represents the characteristic distance from the center at which the curvature becomes effective with regard to the process in the reaction zone and to the propagation velocity.

In the case of *non-similarity* ($L \neq 1$), propagation takes place with a variable temperature at the front and with the attendant steep change in the reaction rate, dictated by the large value of E/R^0T_b. As will be shown later, the corresponding characteristic distance as above is not l_T, but $(E/R^0T_b)l_T$.

Propagation of a laminar flame is described by a set of equations referring, respectively, to the diffusion of the reactive medium, the thermal conduction, the state of the gas (reduced—on account of the low propagation velocity relative to that of sound—to the requirement of dynamic incompressibility), continuity, and momentum. With the dimensionless variables appropriately chosen, the above equations may be formulated as follows:

Diffusion:

$$\rho\frac{\partial C}{\partial t}+\rho U\frac{\partial C}{\partial r}=\frac{1}{L}\frac{1}{r^2}\frac{\partial}{\partial r}\kappa(T)r^2\frac{\partial C}{\partial r}-W(C,T) \quad (1.1)$$

Heat conductivity:

$$\rho\frac{\partial T}{\partial t}+\rho U\frac{\partial T}{\partial r}=\frac{1}{r^2}\frac{\partial}{\partial r}\kappa(T)r^2\frac{\partial T}{\partial r}+(1-\varepsilon)W(C,T) \quad (1.2)$$

Continuity and dynamic incompressibility:

$$\frac{\partial\rho}{\partial t}+\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\rho U)=0, \qquad \rho=\frac{1}{T}. \quad (1.3)$$

The reaction rate W, C, and T are interrelated through Arrhenius' equation:

$$W=z(T)AN^{1+n}\rho^nC^n\exp N\left(1-\frac{1}{T}\right) \quad (1{\cdot}4)$$

where the dimensionless constant A is determined by the parameters of the one-dimensional steady flame:

$$A=z_bnC_0^{n-1}\rho_b\kappa_bc_p^{-1}U_b^{-2}N^{-1-n}\exp(-N). \quad (1.5)$$

Here z_b is the frequency factor at T_b; n is the order of the chemical reaction; and N is the dimensionless activation energy determined as E/R^0T_b. In the theory of flame propagation, the stipulation is always $N \gg 1$, i.e. strong temperature dependence of the reaction rate. In that case, as will be shown in Section 4, A is of the order of unity relative to N, and the width of the reaction zone—of the order of $1/N$ relative to l_T. In other words, the velocity U_b is smaller than its counterpart at zero activation energy and absolute temperature of the unburned gas [$z(0) = 0$] by a factor equal to $N^{(1+n)/2}\exp\frac{1}{2}N$. If however, the above stipulation is dispensed with, the reaction must be assumed to be practically at a standstill. This, in turn, rules out one-dimensional *steady* propagation.

2. DERIVATION OF EQUATION OF MOTION

As point of reference in time ($t = 0$), we take the moment of focussing at which the front radius r_f vanishes. The time preceding this moment is taken as negative.

As is known [2], the width of the reaction zone is of the order $1/N$ relative to l_T. Accordingly, at distances $|r-r_f| \gg 1/N$, the latter may be regarded as a point source, and the velocity W in equations (1.1) and (1.2) may be replaced by a δ-function of strength Q:

$$W(C,T)\to Q\delta(r-r_f). \quad (2.1)$$

As will be shown in Section 4 (see also [10]), the following asymptotic representation of Q holds at the zeroth approximation relative to $1/N$:

$$Q\simeq\exp\tfrac{1}{2}N(T-1). \quad (2.2)$$

In the sequel, we shall conveniently introduce a coordinate system (x, t) linked with the flame front and l_T:

$$x=r-r_f. \quad (2.3)$$

Insofar as the characteristic distance (at which the front curvature becomes effective) is of the order N, we introduce new variables:

$$\Phi=\frac{r_f}{N}; \qquad \tau=\frac{t}{N}. \quad (2.4)$$

The correctness of chosen scales of stretching will be verified *a posteriori* through the requirements of asymptotic matching.

In conjunction with (2.1)–(2.4), the set (1.1)–(1.3) becomes:

$$\frac{1}{N}\rho\frac{\partial T}{\partial\tau}-\rho\frac{\mathrm{d}\Phi}{\mathrm{d}\tau}\frac{\partial T}{\partial x}+\rho U\frac{\partial T}{\partial x}=\left(1+\frac{x}{N\Phi}\right)^{-2}\frac{\partial}{\partial x}\times\kappa\left(1+\frac{x}{N\Phi}\right)^2\frac{\partial T}{\partial x}+(1-\varepsilon)\exp[\tfrac{1}{2}N(T-1)]\delta(x) \quad (2.5)$$

$$\frac{1}{N}\rho\frac{\partial C}{\partial\tau}-\rho\frac{d\Phi}{d\tau}\frac{\partial C}{\partial x}+\rho U\frac{\partial C}{\partial x}=\frac{1}{L}\left(1+\frac{x}{N\Phi}\right)^{-2}\times\frac{\partial}{\partial x}\kappa\left(1+\frac{x}{N\Phi}\right)^{2}\frac{\partial C}{\partial x}-\exp[\tfrac{1}{2}N(T-1)]\delta(x) \quad (2.6)$$

$$\frac{1}{N}\frac{\partial\rho}{\partial\tau}-\frac{d\Phi}{d\tau}\frac{\partial\rho}{\partial x}+\left(1+\frac{x}{N\Phi}\right)^{-2}\frac{\partial}{\partial x}\left(1+\frac{x}{N\Phi}\right)^{2}\rho U=0$$

$$\rho=\frac{1}{T} \quad (2.7)$$

whose solution is sought in the form of the following asymptotic expansions in $(1/N)$:

$$\left.\begin{array}{l} C=C^0(x,\tau)+\frac{1}{N}C^1(x,\tau)+\ldots \\ \qquad \text{where} \quad C(x,\tau)=0 \text{ for } x>0 \\ T=T^0(x,\tau)+\frac{1}{N}T^1(x,\tau)+\ldots \\ \qquad \text{where} \quad T^0(x,\tau)=1 \text{ for } x>0 \\ U=U^0(x,\tau)+\frac{1}{N}U^1(x,\tau)+\ldots \\ \rho=\rho^0(x,\tau)+\frac{1}{N}\rho^1(x,\tau)+\ldots \\ \Phi=\Phi^0(\tau)+\frac{1}{N}\Phi^1(\tau)+\ldots. \end{array}\right\} \quad (2.8)$$

Equations (2.5)–(2.8) yield, for the zeroth approximation, the following set of quasi-stationary equations:

$$\rho^0\left(U^0-\frac{d\Phi^0}{d\tau}\right)\frac{\partial T^0}{\partial x}=\frac{\partial}{\partial x}\kappa^0\frac{\partial T^0}{\partial x}+(1-\varepsilon)\exp\tfrac{1}{2}T^1\delta(x) \quad (2.9)$$

$$\rho^0\left(U^0-\frac{d\Phi^0}{d\tau}\right)\frac{\partial C^0}{\partial x}=\frac{1}{L}\frac{\partial}{\partial x}\kappa^0\frac{\partial C^0}{\partial x}-\exp\tfrac{1}{2}T^1\delta(x) \quad (2.10)$$

$$\frac{\partial}{\partial x}\left(\rho U^0-\rho^0\frac{d\Phi^0}{d\tau}\right)=0;\ \rho^0=\frac{1}{T^0};\ \kappa^0=\kappa(T^0) \quad (2.11)$$

which, solved subject to the boundary conditions:

$$\left.\begin{array}{l} T^0(-\infty,\tau)=\varepsilon;\ C^0(-\infty,\tau)=1;\ U^0(-\infty,\tau)=0 \\ T^0(0,\tau)=1; \quad C^0(0,\tau)=0; \end{array}\right\} \quad (2.12)$$

yield in turn for $x<0$

$$\left.\begin{array}{l} -\frac{1}{\varepsilon}\frac{d\Phi^0}{d\tau}(T^0-\varepsilon)=\kappa^0\frac{\partial T^0}{\partial x} \\ C^0=1-\left(\frac{T^0-\varepsilon}{1-\varepsilon}\right)^L \\ U^0=-\frac{1}{\varepsilon}\frac{d\Phi^0}{d\tau}(T^0-\varepsilon). \end{array}\right\} \quad (2.13)$$

In addition, integrating (2.9) near $x=0$, we find:

$$T^1(0,\tau)=2\ln\left(-\frac{1}{\varepsilon}\frac{d\Phi^0}{d\tau}\right). \quad (2.14)$$

In order to find the second equation linking $T^1(0,\tau)$ and $\Phi^0(\tau)$, we need information on the next approximation relative to $1/N$. To this end, we use the linear combination of (2.9) and (2.10), obtained by eliminating the reaction rate:

$$\frac{1}{N}\rho\frac{\partial}{\partial\tau}(T+(1-\varepsilon)C)+\rho\left(U-\frac{d\Phi}{d\tau}\right)\frac{\partial}{\partial x}(T+(1-\varepsilon)C)=\left(1+\frac{x}{N\Phi}\right)^{-2}\frac{\partial}{\partial x}\kappa\left(1+\frac{x}{N\Phi}\right)^{2}\frac{\partial}{\partial x}\times\left(T+\frac{1-\varepsilon}{L}C\right). \quad (2.15)$$

At the first approximation, (2.15) and (2.7) have the form:

$$\rho^0\frac{\partial}{\partial\tau}(T^0+(1-\varepsilon)C^0)\left(\rho^0U^1+\rho^1U^0-\rho^0\frac{d\Phi^1}{d\tau}-\rho^1\frac{d\Phi^0}{d\tau}\right)\frac{\partial}{\partial x}\times(T^0+(1-\varepsilon)C^0)+\rho^0\left(U^0-\frac{d\Phi^0}{d\tau}\right)\times\frac{\partial}{\partial x}(T^1+(1-\varepsilon)C^1)$$
$$=\frac{\partial}{\partial x}\left(\frac{d\kappa^0}{dT^0}T^1\frac{\partial}{\partial x}\left(T^0+\frac{1-\varepsilon}{L}C^0\right)+\kappa^0\frac{\partial}{\partial x}\left(T^1+\frac{1-\varepsilon}{L}C^1\right)\right)+\frac{2\kappa^0}{\Phi^0}\frac{\partial}{\partial x}\left(T^0+\frac{1-\varepsilon}{L}C^0\right) \quad (2.16)$$

$$\left.\begin{array}{l} \frac{\partial\rho^0}{\partial\tau}+\frac{\partial}{\partial x}\left(U^0\rho^1-\frac{d\Phi^0}{d\tau}\rho^1+U^1\rho^0-\frac{d\Phi^1}{d\tau}\rho^0\right)+\frac{2\rho^0U^0}{\Phi^0}=0 \\ \rho^1=-\frac{T^1}{T^{02}}. \end{array}\right\} \quad (2.17)$$

Bearing in mind (2.8), equation (2.16) yields for $x>0$

$$T^1(x,\tau)=T^1(0,\tau). \quad (2.18)$$

Integrating (2.16) with respect to x in the $(-\infty,0)$ interval, and bearing in mind (2.18), (2.13), and the condition $T^1(-\infty,\tau)=C^1(-\infty,\tau)=0$, we obtain:

$$\begin{aligned}\frac{\partial}{\partial \tau}\int_{-\infty}^{0} & \rho^0(T^0+(1-\varepsilon)C^0-1)\,\mathrm{d}x \\ & +\frac{2}{\Phi^0}\int_{-\infty}^{0}\rho^0U^0(T^0+(1-\varepsilon)C^0-1)\,\mathrm{d}x \\ & -\frac{1}{\varepsilon}\frac{\mathrm{d}\Phi^0}{\mathrm{d}\tau}T^1(0,\tau) \\ & =\frac{2}{\Phi^0}\int_{-\infty}^{0}\kappa^0\frac{\partial}{\partial x}\left(T^0+\frac{1-\varepsilon}{L}C^0\right)\mathrm{d}x \qquad (2.19)\end{aligned}$$

$T^1(0,\tau)$ is thus expressed through the zeroth approximation of the problem. Using (2.13), we convert from x to T^0 as variable of integration, whereby (2.19) (2.19) becomes:

$$\begin{aligned}\varepsilon\frac{\mathrm{d}^2\Phi^0}{\mathrm{d}\tau^2}\left(\frac{\mathrm{d}\Phi^0}{\mathrm{d}\tau}\right)^{-2} & \int_{\varepsilon}^{1}\frac{\kappa(T)}{T}\left[1-\left(\frac{T-\varepsilon}{1-\varepsilon}\right)^{L-1}\right]\mathrm{d}T \\ & -\frac{1}{\varepsilon}\frac{\mathrm{d}\Phi^0}{\mathrm{d}\tau}T^1(0,\tau) \\ & =\frac{2\varepsilon}{\Phi^0}\int_{\varepsilon}^{1}\frac{\kappa(T)}{T}\left[1-\left(\frac{T-\varepsilon}{1-\varepsilon}\right)^{L-1}\right]\mathrm{d}T \qquad (2.20)\end{aligned}$$

so that, in conjunction with (2.14), we have a non-linear differential equation for Φ^0.

Introducing new variables G and ξ,

$$\begin{aligned}\Phi^0 &= G\varepsilon\int_{\varepsilon}^{1}\frac{\kappa(T)}{T}\left[1-\left(\frac{T-\varepsilon}{1-\varepsilon}\right)^{L-1}\right]\mathrm{d}T \\ \tau &= -\xi\int_{\varepsilon}^{1}\frac{\kappa(T)}{T}\left[1-\left(\frac{T-\varepsilon}{1-\varepsilon}\right)^{L-1}\right]\mathrm{d}T\end{aligned} \qquad (2.21)$$

we have by (2.14):

$$T^1(0,\tau)=2\ln\frac{\mathrm{d}G}{\mathrm{d}\xi} \qquad (2.22)$$

and by (2.20)–(2.22):

$$\frac{\mathrm{d}^2G}{\mathrm{d}\xi^2}+2\left(\frac{\mathrm{d}G}{\mathrm{d}\xi}\right)^3\ln\frac{\mathrm{d}G}{\mathrm{d}\xi}=\frac{2}{G}\left(\frac{\mathrm{d}G}{\mathrm{d}\xi}\right)^2. \qquad (2.23)$$

The integral in (2.21) changes sign at $L = 1$. For $\tau < 0$, as was already noted, $\xi > 0$ corresponds to the solution for $L > 1$, and $\xi < 0$ – to that for $L < 1$. Denoting:

$$\frac{\mathrm{d}G}{\mathrm{d}\xi}=P(G). \qquad (2.24)$$

Equation (2.23) is a first-order equation in $P(G)$, namely:

$$\frac{\mathrm{d}P}{\mathrm{d}G}=\frac{2P(1-GP\ln P)}{G}. \qquad (2.25)$$

Only solutions for which $\mathrm{d}\Phi^0/\mathrm{d}\tau$ is negative, i.e. $P > 0$, have a physical meaning.

In the (G, P)-plane, the integral curves of equation (2.25) from the schematic pattern shown in Fig. 2, with the arrows indicating positive time. At $\tau\to\infty(G\to\pm\infty)$, the flame front must tend to normal propagation, for which $P = 1$. This requirement is the boundary condition of the problem.

3. BEHAVIOR OF SOLUTIONS OF (2.25)

Near the point $P = 1$, $G = \pm\infty$, the following asymptotics is valid for the solution of (2.25):

$$P\simeq 1+\frac{1}{G}. \qquad (3.1)$$

As can be seen from Fig. 2, in the region $G < 0 (L < 1)$, a single monotonically-descending curve runs from $P = 1, G = -\infty$ to $P = 0$, $G = 0 (P \sim G^2)$. Accordingly,

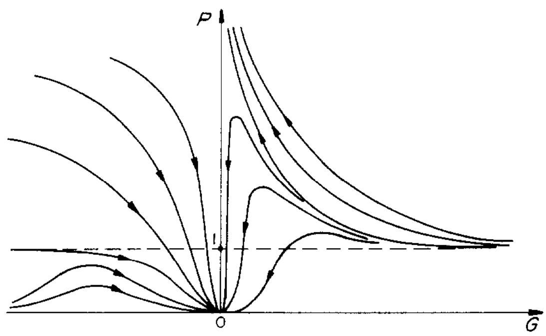

FIG. 2. Plane of equation (2.25).

as the flame front approaches the center, its propagation velocity decreases. As is seen from equation (2.2), the temperature at the front also decreases monotonically to $-\infty$. However, as is shown in [4] (see also [2]), for the flame to be capable of propagation, the temperature drop in the reaction zone cannot exceed $\sim RT_b^2/E$, otherwise the flame is extinguished.

In the region $G > (L > 1)$, the picture is different: a whole family of curves originates at $P = 1$, $G = +\infty$; part of them, monotonically ascending, terminate at $P = +\infty$, $G = +0[G \sim 2/(P\ln P)]$. At first glance, all these curves have a physical meaning, implying existence of an infinite number of solutions, the propagation velocity of each of which increases to $+\infty$ together with the temperature at the front. This circumstance (just as in the case $L < 1$) reflects the intermediate character of the obtained asymptotics [3]. It should be noted that the infinite set of solutions contains one corresponding to a minimal propagation velocity for every $r(G)$. As can be seen from the figure, this solution is the *envelope* of the curves for which $P \to 0$ at $G \to 0$.

Similar non-uniqueness, associated with a minimal propagation velocity, is discussed in [5] and [6] which deal respectively with wave propagation in a chain reaction and with a converging shock wave. In our own case, the hypothesis of realization of the minimal propagation velocity may be verified through numerical analysis, using initial data.

4. VERIFICATION OF ASYMPTOTIC RELATIONSHIP (2.2)

Equations (2.9) and (2.10) integrated near $x = 0$ yield:

$$\frac{\partial T^0(-0,\tau)}{\partial x} = (1-\varepsilon)\exp\tfrac{1}{2}T^1(0,\tau) \qquad (4.1)$$

$$\frac{1}{L}\frac{\partial C^0(-0,\tau)}{\partial x} = -\exp\tfrac{1}{2}T^1(0,\tau). \qquad (4.2)$$

The above relationships follow from the assumed asymptotic relationship (2.2). We shall show that they are a rigorous consequence of the matching conditions for the respective asymptotic solutions inside and outside the reaction zone; by this means, relationship (2.2) will be verified at the zeroth approximation relative to $1/N$. Our point of departure is the set of diffusion and conduction equations with distributed heat and mass sources. To this end, the point source in (2.5)–(2.7) must be replaced by a corresponding reaction rate W:

$$\exp\tfrac{1}{2}T^1\delta(x) \to W(C,T). \qquad (4.3)$$

Insofar as the width of the reaction zone is of order $1/N$ relative to l_T, we take as the characteristic space variable for the reaction zone:

$$X = xN. \qquad (4.4)$$

In this region, the asymptotic expansions for temperature, concentration, and the constant A are sought in the form:

$$\left.\begin{aligned} T &= 1+\frac{1}{N}\theta(X,\tau)+\ldots \\ C &= \frac{1}{N}S(X,\tau)+\ldots \\ A &= A^0+\ldots. \end{aligned}\right\} \qquad (4.5)$$

The set (2.5)–(2.7), modified as per (4.3)–(4.5) for the zeroth approximation relative to $1/N$, yields:

$$\left.\begin{aligned} &\frac{\partial^2\theta}{\partial X^2}+(1-\varepsilon)A^0S^n\exp\theta = 0 \\ &\frac{1}{L}\frac{\partial^2 S}{\partial X^2}-A^0S^n\exp\theta = 0. \end{aligned}\right\} \qquad (4.6)$$

Insofar as $T^0(x,\tau) = 1$, $C^0(x,\tau) = 0$ for $x > 0$, the matching conditions read:

$$\theta(+\infty,\tau) = T^1(0,\tau) \qquad (4.7)$$

$$\frac{\partial\theta(-\infty,\tau)}{\partial X} = \frac{\partial T^0(-0,\tau)}{\partial x} \qquad (4.8)$$

$$S(+\infty,\tau) = 0; \qquad \frac{\partial S(-\infty,\tau)}{\partial X} = \frac{\partial C^0(-0,\tau)}{\partial x}. \qquad (4.9)$$

Equation (4.6), in conjunction with (4.7) and (4.9), yields:

$$\theta(X,\tau)+\frac{1-\varepsilon}{L}S(X,\tau) = T^1(0,\tau) \qquad (4.10)$$

and may be reduced to a single equation in θ.

$$\frac{\partial^2\theta}{\partial X^2}+\frac{A^0L^n}{(1-\varepsilon)^{n-1}}(T^1-\theta)^n\exp\theta = 0 \qquad (4.11)$$

$$\left(\frac{\partial\theta}{\partial X}\right)^2-\frac{2A^0L^n}{(1-\varepsilon)^{n-1}}\int_{T_1}^{\theta}(T^1-\theta)^n\exp\theta\,\mathrm{d}\theta \qquad (4.12)$$

and

$$\left(\frac{\partial\theta(-\infty,\tau)}{\partial X}\right)^2 = \frac{2A^0L^n\exp T^1}{(1-\varepsilon)^{n-1}}\int_{-\infty}^{0}(-Z)^n\exp Z\,\mathrm{d}z = \frac{2A^0n!L^n\exp T^1(0,\tau)}{(1-\varepsilon)^{n-1}}. \qquad (4.13)$$

By (2.13), we have for normal propagation:

$$-\frac{1}{\varepsilon}\frac{\mathrm{d}\Phi^0}{\mathrm{d}\tau} = 1; \quad \frac{\partial T^0(-0,\tau)}{\partial x} = 1-\varepsilon; \quad T^1(0,\tau) = 0 \qquad (4.14)$$

and equations (4.8), (4.13), and (4.14) yield:

$$A^0 = \frac{(1-\varepsilon)^{1+n}}{2L^n n!} \qquad (4.15)$$

which represents the principal term for the asymptotics of U_b (1.5).

Finally, by (4.13) and (4.15):

$$\frac{\partial\theta(-\infty,\tau)}{\partial X} = (1-\varepsilon)\exp\tfrac{1}{2}T^1(0,\tau) \qquad (4.16)$$

which, in accordance with (4.8), is equivalent to the sought relationship (4.1). Equation (4.2) is an elementary consequence of (4.10) and (4.16).

5. ON THE INSTABILITY OF NORMAL FLAME PROPAGATION

The asymptotic approach developed above may be applied to the one-dimensional-flat flame equations to examine the stability of its normal propagation. In the case of similarity between the temperature and concentration fields, i.e. at $L = 1$, normal propagation is steady—a fact established by several authors using various analytical methods [7–9]. At $L \neq 1$, however, the corresponding mathematical problem is seriously complicated. It appears that under $L > 1$ and $N \gg 1$ the class of one-dimensional perturbation may be found, in relation with which the normal flame propagation is unstable.

The subsequent numerical analysis shows the possibility of a steady self-oscillatory regime setting in rather than normal propagation. In the case of plane one-dimensional problem, the set (1·1–3) becomes:

Diffusion:

$$\rho\frac{\partial C}{\partial t}+\rho U\frac{\partial C}{\partial r}=\frac{1}{L}\frac{\partial}{\partial r}\kappa(T)\frac{\partial C}{\partial r}-W(C,T) \quad (5.1)$$

Heat conductivity:

$$\rho\frac{\partial T}{\partial t}+\rho U\frac{\partial T}{\partial r}=\frac{\partial}{\partial r}\kappa(T)\frac{\partial T}{\partial r}+(1-\varepsilon)W(C,T) \quad (5.2)$$

Continuity and dynamic incompressibility:

$$\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial r}\rho U=0 \qquad \rho=\frac{1}{T}. \quad (5.3)$$

Let the flame propagating in the negative direction of axe r, where $r = 0$, corresponds to the flame front position at the moment of initial perturbation $t = 0$. Let us introduce a coordinate x, connected with the flame front and stretched variables Φ and τ, (see 2.3 and 2.4) r_f = flame front position at moment t.

In conjunction with (2.1)–(2.4), the set (5.1)–(5.3) becomes:

$$\frac{1}{N}\rho\frac{\partial T}{\partial \tau}-\rho\frac{\mathrm{d}\Phi}{\mathrm{d}\tau}\frac{\partial T}{\partial x}+\rho U\frac{\partial T}{\partial x}=\frac{\partial}{\partial x}\kappa\frac{\partial T}{\partial x}+(1-\varepsilon)\exp[\tfrac{1}{2}N(T-1)]\delta(x) \quad (5.4)$$

$$\frac{1}{N}\rho\frac{\partial C}{\partial \tau}-\rho\frac{\mathrm{d}\Phi}{\mathrm{d}\tau}\frac{\partial C}{\partial x}+\rho U\frac{\partial C}{\partial x}=\frac{1}{L}\frac{\partial}{\partial x}\kappa\frac{\partial C}{\partial x}-\exp[\tfrac{1}{2}N(T-1)]\delta(x) \quad (5.5)$$

$$\frac{1}{N}\frac{\partial \rho}{\partial \tau}-\frac{\mathrm{d}\Phi}{\mathrm{d}\tau}\frac{\partial \rho}{\partial x}+\frac{\partial}{\partial x}\rho U=0, \qquad \rho=\frac{1}{T}. \quad (5.6)$$

Hence, after the treatments, like those undertaken in Section 2, for the front flame coordinate in (P, G) plane, the equation yields:

$$\frac{\mathrm{d}P}{\mathrm{d}G}=-2P\ln P \quad (5.7)$$

where P and G are determined in (2.21) and (2.24). The straight line $P = 1$ corresponds to the normal flame propagation.

$$\frac{\mathrm{d}P}{\mathrm{d}G}>0 \quad \text{under} \quad 0<P<1$$

$$\frac{\mathrm{d}P}{\mathrm{d}G}<0 \quad \text{under} \quad P>1.$$

In the case of $L < 1$, with the time growing, the integral curves tend to the line $P = 1$. At $L > 1$, with the time growing, the integral curves of the equation (5.7) get away from $P = 1$, which indicates instability of the normal flame propagation. The equation (5.7) describes the asymptotic solution of set (5.1)–(5.3) at $N \to \infty$, and it is right only for a limited space of time ($t \sim N$).

In order to clarify the subsequent development of the propagations, we shall solve numerically the problem of steady propagation triggered by an ignition source.

Let, for the sake of simplicity, the initial ignition source be symmetrical with respect to $r = 0$. Then the convective terms in the equations (5.1)(5.2) may be removed by the transition to new coordinates (Lagrangian).

$$\int_0^r \rho(x,\tau)\,\mathrm{d}x=\zeta. \quad (5.8)$$

In this new system of coordinates, the set (5.4)–(5.6) becomes:

$$\frac{\partial T}{\partial \tau}=\frac{\partial}{\partial \zeta}\lambda\frac{\partial T}{\partial \zeta}+(1-\varepsilon)F; \qquad \frac{\partial C}{\partial \tau}=\frac{1}{L}\frac{\partial}{\partial \zeta}\lambda\frac{\partial C}{\partial \zeta}-F$$
$$\text{where} \quad \lambda=\kappa/T, \quad F=TW. \quad (5.9)$$

The problem is solved in the half space $-\infty < \zeta < 0$, the solution being sought under the following initial and boundary conditions:

$$\begin{aligned}&\text{For } -0{\cdot}5<\zeta<0: && T(0,\zeta)=1\\ &\text{for } -\infty<\zeta<-0{\cdot}5: && T(0,\zeta)=\varepsilon\\ &\text{For } -\infty<\zeta<0: && C(0,\zeta)=1\\ & && \frac{\partial C}{\partial \zeta}(\tau,0)=\frac{\partial T}{\partial \zeta}(\tau,0)=0.\end{aligned} \quad (5.10)$$

The set (5.9) and (5.10) is solved in a quasi-explicit difference scheme of the "tripod" type, with steps constant both in time and over the half-space and related through Courant's stability condition:

$$(\Delta\tau)=\tfrac{1}{2}(\Delta\zeta)^2\sqrt{(\varepsilon)}.$$

For $\kappa = \sqrt{(T)}$, $n = 1$, the scheme has the form:

$$\frac{T_j^{k+1}-T_j^k}{(\Delta\tau)}=\frac{1}{(\Delta\zeta)^2}\left(\frac{T_{j+1}^k-T_j^k}{\sqrt{(T_j^k)}}-\frac{T_j^k-T_{j-1}^k}{\sqrt{(T_{j-1}^k)}}\right)+(1-\varepsilon)F_{jk}$$

$$\frac{C_j^{k+1}-C_j^k}{(\Delta\tau)}=\frac{1}{L(\Delta\zeta)^2}\left(\frac{C_{j+1}^k-C_j^k}{\sqrt{(T_j^k)}}-\frac{C_j^k-C_{j-1}^k}{\sqrt{(T_{j-1}^k)}}\right)-F_{jk}$$

where $F_{jk}=N^2C_j^{k+1}\exp N(1-(1/T_j^k))$; $j = 1, 2, 3, \ldots$ (j = corresponding to $\zeta = 0$); and $k = 1, 2, 3, \ldots$ ($k = 1$ corresponding to $\tau = 0$).

The numerical calculation showed (Fig. 3) that for

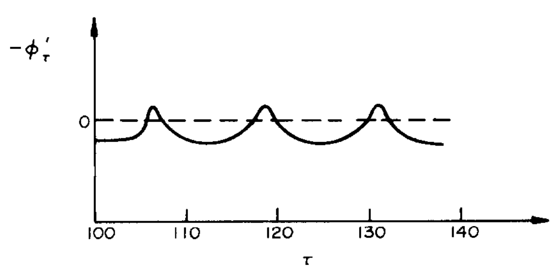

FIG. 3. Flame front velocity as a function of time ($\varepsilon = 0{\cdot}25$, $N = 30$, $L = 2$).

$L > 1$ and sufficiently large N, a self-oscillating regime sets in rather than steady propagation.

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SUR UN FRONT DE FLAMME SPHERIQUE ET CONVERGENT

Résumé—L'article traite de la vitesse de propagation frontale d'une flamme à symétrie sphérique qui converge vers le centre. Ce genre de problème peut décrire la combustion dans une zone désunie dans la théorie laminale d'une flamme turbulente. On montre que pour un nombre de Lewis supérieur à l'unité, la température dans la zone de réaction augmente, ainsi que la vitesse de propagation, lorsque le front s'approche du centre. Au contraire, quand le nombre de Lewis est inférieur à l'unité, la propagation est ralentie et la température dans la zone de réaction conduit à l'extinction.

Le problème est traité avec l'hypothèse d'une forte dépendance de la vitesse de réaction vis à vis de la température.

L'approche asymptotique développée en résolvant ce problème peut être appliquée à l'étude de la stabilité de la propagation d'une flamme. On montrerait que la propagation est instable pour un nombre de Lewis supérieur à l'unité et qu'il s'instaure un régime oscillatoire.

DAS VERHALTEN EINER KUGELFÖRMIGEN FLAMMENFRONT

Zusammenfassung—Die Arbeit befaßt sich mit der frontalen Ausbreitungsgeschwindigkeit einer kugelsymmetrischen Flamme, die in einem Zentrum zusammenläuft. Ein Problem dieser Art kann mit Hilfe der Schichtentheorie für turbulente Flammen beschrieben werden. Es wird gezeigt, daß für Lewis-Zahlen größer 1 die Temperatur in der Reaktionszone und die Ausbreitungsgeschwindigkeit ansteigt, wenn die Flammenfront sich dem Zentrum nähert. Im Gegensatz hierzu verläuft die Ausbreitung langsamer bei bis zum Erlöschen steigender Temperatur in der Reaktionszone für Lewis-Zahlen kleiner 1. Bei der Lösung des Problems wird angenommen, daß die Reaktionsrate stark von der Temperatur abhängt. Die entwickelte Näherungslösung kann bei der Untersuchung der Stabilität normaler Flammenausbreitung angewendet werden. Der Artikel soll zeigen, daß bei Lewis-Zahlen größer 1 die Ausbreitung instabil wird und Eigenschwingungsverhalten einsetzt.

О СУЖАЮЩЕМСЯ СФЕРИЧЕСКОМ ФРОНТЕ ПЛАМЕНИ

Аннотация — В статье рассматривается фронтальная скорость распространения сферически симметричного пламени, сходящегося в фокус. С помощью этой задачи может быть описан процесс горения в разъединенной зоне в теории расслоения турбулентного пламени. Показано, что если число Льюиса больше единицы, то температура в зоне реакции возрастает вместе со скоростью распространения пламени по мере приближения фронта к фокусу. И наоборот, если

число Льюиса меньше единицы, то распространение замедляется, а температура в зоне реакции падает до затухания.

Задача решена при условии, что скорость реакции сильно зависит от температуры.

Асимптотический метод, разработанный при решении этой задачи, может быть использован для изучения устойчивости нормальной скорости распространения пламени. Авторы статьи хотят показать, что при числе Льюиса больше единицы распространение пламени является неустойчивым и возникает режим автоколебаний.